

Mass and Total Energy of Moving Bodies In a Quantified Expansion

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Abstract. In this paper, we present a new formulation of Lorentz transformations using a metric that quantifies the space expansion. As a consequence, we sort out that the limiting velocity of moving bodies is decreasing together with the space expansion. A new adjustment of relativistic laws is added to incorporate the non static nature of space-time. The conservation of the physical laws at each step of the quantified expansion allows the obtaining of new formalisms for the physical laws, in particular when an object starts moving under any force, its total energy, momentum and mass are directly affected by the expansion of the space. An example of inelastic collision is studied and several conclusions derived, specially the example of fission of atoms leads to clear correlation between liberated energy and universe expansion, it turns out that the liberated energy is increasing together with the universe expansion.

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1 Introduction

1.1 Plan

In this article we introduce a practical quantification of the space expansion in our universe using a metric with a numerical product that quantifies the expansion of the space step by step (we will call it universe expansion).

In the second section we rewrite the Lorentz transformation equations using the new metric to obtain a family of Lorentz transformation equations $(T_n)_{n \geq 0}$, where each transformation T_n represents the Lorentz transformation equations at the step n of the space expansion. This family of transformation leads to a decreasing limiting velocity of moving bodies from one step to another, and then the physical laws are affected by the universe expansion, followed by conclusions derived concerning energy, masses, and momentum in an expanding space.

In section three, the composition of velocity is derived using T_n -Lorentz transformation equations to discuss the invariance of the new limiting velocities of matter.

The rest mass of photon is then discussed in section forth and new quantum formalism is introduced. We conclude this section by discussing an inelastic collision in which we find that the conversion of the rest-energy into kinetic energy is affected by the universe expansion as well as the fact that the total liberated energy by fission of atoms is increasing naturally together with the universe expansion. We conclude by some comments related to the notion of time and proper time.

1.2 Expansion quantification

From the rate at which galaxies are receding from each other in our universe, it can be deduced that all galaxies must have been very close to each other at the same time in the past. Meanwhile galaxies are receding from each other what really happen to the physical laws, are they affected by the process of expansion or not? In the purpose to investigate the status of the physical laws together with the universe expansion we propose the following quantification of the universe movement:

Suppose that we can quantify the whole universe movement, from the beginning of the universe expansion where matter was together until present time with current picture of matter distribution. Suppose that we can subdivide the whole process of expansion from the beginning until present time into n steps, in which we approximate the rate of change of the distance between any two separated events from one step to another together with the universe expansion as follow:

Quantification

Q: if the distance between two events at the step n is equal to L_n , and at the step $(n+1)$ is equal to L_{n+1} then $L_{n+1} = a_{n+1}L_n$ for all $n \geq 0$, where a_n is a sequence such that $a_0 = 1$, $a_n > 1 \forall n \geq 1$, and $\prod_{i=0}^n a_i$ converges.

For this quantification the step 0 represents the beginning of the expansion of our universe (once $L_0 \neq 0$) and the space-time is defined as the set of events together with the notion of squared interval S_0^2 defined between any two events. An event A in a reference frame \mathcal{S} is fixed by four coordinates x_0, y_0, z_0 and t_0 , where (x_0, y_0, z_0) are the spatial coordinates of the point in space when the event in question occurs at the step 0, and where t_0 fixes the instant when this event occurs. Another event B will be described within the same reference frame by four different coordinates $x_0 + \Delta x_0, y_0 + \Delta y_0, z_0 + \Delta z_0$, and $t_0 + \Delta t_0$. The points in space where these two events occur are separated by the distance L_0 given by $L_0^2 = \Delta^2 x_0 + \Delta^2 y_0 + \Delta^2 z_0$. The moments in time when these two events occur are separated at the step 0 by a time interval given by Δt_0 . The squared interval S_0^2 between these two events is given as function of these quantities, by definition, through the generalization of Pythagorean Theorem

$$S_0^2 = c^2 \Delta^2 t_0 - L_0^2 = c^2 \Delta^2 t_0 - (\Delta^2 x_0 + \Delta^2 y_0 + \Delta^2 z_0) \quad (1)$$

where c is the maximum speed of signal propagation at the beginning of the universe expansion (the maximum speed for the transmission of information).

At the step 1 of the universe expansion, the event A in the reference frame \mathcal{S} is fixed by four new coordinates (x_1, y_1, z_1) and t_1 , where (x_1, y_1, z_1) are the spatial coordinates of the point in space when the event in question occurs at the step 1, and where t_1 fixes the instant when this event occurs, meanwhile the event B is described within the same reference frame by four different coordinates $x_1 + \Delta x_1, y_1 + \Delta y_1, z_1 + \Delta z_1$, and $t_1 + \Delta t_1$. The points in space where these two events occur are separated by the distance L_1 given by $L_1^2 = \Delta^2 x_1 + \Delta^2 y_1 + \Delta^2 z_1$. While the universe expands from the step 0 to the step 1 at a rate such that $\Delta x_1 = a_1 \Delta x_0, \Delta y_1 = a_1 \Delta y_0$ and $\Delta z_1 = a_1 \Delta z_0$, then we

have $L_1 = a_1 L_0$. The moments in time when these two events occur are separated at the step 1 by a time interval given by Δt_1 . The squared interval S_1^2 between these two events is given as function of these quantities, by definition, through the generalization of Pythagorean Theorem

$$S_1^2 = c^2 \Delta^2 t_1 - L_1^2 = c^2 \Delta^2 t_1 - a_1^2 (\Delta^2 x_0 + \Delta^2 y_0 + \Delta^2 z_0) \quad (2)$$

where c is the maximum speed of signal propagation at the beginning of the universe expansion, etc..

At the step n of the universe expansion, the event A in the reference frame \mathcal{S} is fixed by four coordinates x_n, y_n, z_n and t_n , where (x_n, y_n, z_n) are the spatial coordinates of the point in space when the event in question occurs at the step n , and where t_n fixes the instant when this event occurs, meanwhile the event B is described within the same reference frame by four different coordinates $x_n + \Delta x_n, y_n + \Delta y_n, z_n + \Delta z_n$, and $t_n + \Delta t_n$. The points in space where these two events occur are separated by the distance L_n given by $L_n^2 = \Delta^2 x_n + \Delta^2 y_n + \Delta^2 z_n$. While the universe expands from the step $(n-1)$ to the step n at a rate such that $\Delta x_n = a_n \Delta x_{n-1}$, $\Delta y_n = a_n \Delta y_{n-1}$ and $\Delta z_n = a_n \Delta z_{n-1}$ then we have

$$L_n = a_n L_{n-1} = a_n a_{n-1} L_{n-2} = \dots = \left(\prod_{i=1}^n a_i \right) L_0. \quad (3)$$

The moments in time when these two events occur are separated at the step n by a time interval given by Δt_n . The squared interval S_n^2 between these two events is given as function of these quantities, by definition, through the generalization of Pythagorean Theorem

$$S_n^2 = c^2 \Delta^2 t_n - L_n^2 = c^2 \Delta^2 t_n - \left(\prod_{i=1}^n a_i \right)^2 L_0 \quad (4)$$

which gives

$$S_n^2 = c^2 \Delta^2 t_n - \left(\prod_{i=1}^n a_i \right)^2 (\Delta^2 x_0 + \Delta^2 y_0 + \Delta^2 z_0) \quad (5)$$

where c is the maximum speed of signal propagation at the beginning of the universe expansion.

The above metrics, from the beginning of the universe expansion (step 0) until the present time, quantify the expansion of the space-time step by step. Indeed, a ball $\mathcal{B}_0(0, r)$ defined by

$$x_0^2 + y_0^2 + z_0^2 \leq r^2 \quad (6)$$

will increase if its radius increases, and we have for $a_i > 1$, $i = 1, 2, \dots, n$, $\forall n \in \mathbb{N}$,

$$\mathcal{B}_0(0, r) \subset \mathcal{B}_1(0, (a_1)r) \subset \mathcal{B}_2(0, (a_1.a_2)r) \subset \dots \subset \mathcal{B}_n(0, (\prod_{i=0}^n a_i)r). \quad (7)$$

However, for all $n \in \mathbb{N}$, the sphere $\mathcal{S}_n(0, (\prod_{i=0}^n a_i)r) \subset \mathcal{B}_n(0, (\prod_{i=0}^n a_i)r)$, is an expanding sphere given by

$$x_n^2 + y_n^2 + z_n^2 = \left(\prod_{i=0}^n a_i^2\right)r^2 = \left(\prod_{i=0}^n a_i^2\right)(x_0^2 + y_0^2 + z_0^2) \quad (8)$$

where $\prod_{i=0}^n a_i > 1$ is an increasing and bounded product.

The infinitesimal space-time interval at the step n , $\forall n \in \mathbb{N}$, between two neighboring events within a locally inertial frame metric takes then the form

$$d\sigma_n^2 = c^2 dt_n^2 - \left(\prod_{i=0}^n a_i\right)^2 (dx^2 + dy^2 + dz^2), \quad (9)$$

in which n represents the expansion step of the physical universe.

In such locally inertial reference frame the laws of the special relativity apply at each step. We propose to rewrite the physical laws in each space-time step by step to investigate any possible evolution in the laws of nature.

Remark 1 *The metric (9) can be written as*

$$d\sigma_n^2 = c^2 dt_n^2 - (dx_n^2 + dy_n^2 + dz_n^2), \quad (10)$$

with $dx_n = (\prod_{i=0}^n a_i)dx_0$, $dy_n = (\prod_{i=0}^n a_i)dy_0$, and $dz_n = (\prod_{i=0}^n a_i)dz_0$.

2 Lorentz transformations equations

Let us consider an expanding space \mathcal{E} in which the proper distance between any distant points is increasing following the above quantification \mathcal{Q} , and let us consider that the expansion is totally modeled by an adequate numerical quantification represented by the product $(\prod_{i=0}^n a_i)$ introduced above (that is to say if the position of a body at rest or the location of an event is expressed locally via three space coordinates (x, y, z) in an inertial frame at the step 0, then at the step 1 its location at rest is expressed in the same frame by (a_1x, a_1y, a_1z) , at the step 2 is expressed in the same frame by $((a_1a_2)x, (a_1a_2)y, (a_1a_2)z)$, and at the step n its location at rest is expressed by $((\prod_{i=0}^n a_i)x, (\prod_{i=0}^n a_i)y, (\prod_{i=0}^n a_i)z)$, where n represents the expansion step of the space movement quantification).

Let us consider two Cartesian frames $S(x, y, z, t)$ and $S'(x', y', z', t')$ in \mathcal{E} such that at each expansion step the equations of Newtonian mechanics hold good. Suppose that an event has Cartesian coordinates (x, y, z, t) relative to S , and (x', y', z', t') relative to S' at the same step of expansion. In general for a linear transformation of the Cartesian frames we have

$$\begin{cases} x' = a_{xx}x + a_{xy}y + a_{xz}z + a_{xt}t \\ y' = a_{yx}x + a_{yy}y + a_{yz}z + a_{yt}t \\ z' = a_{zx}x + a_{zy}y + a_{zz}z + a_{zt}t \\ t' = a_{tx}x + a_{ty}y + a_{tz}z + a_{tt}t \end{cases} \quad (11)$$

where the coefficients a_{ij} depend on the movement. The general properties of homogeneity and isotropy of the space with the choice that the frame S' moves in the x direction of S with uniform velocity v (without losing generality), such that the corresponding axes of S and S' remain parallel throughout the motion having coincided at $t = t' = 0$ will give

$$\begin{cases} x' = a_{xx}(x - vt) \\ y' = a_{yy}y \\ z' = a_{zz}z \\ t' = a_{tx}x + a_{tt}t \end{cases} \quad (12)$$

Suppose that a light signal is emitted from the common center of the two frames at $t = t' = 0$. Since the space \mathcal{E} is expanding, then the observer in the frame S will observe that the light travels in all direction in an expanding sphere, at the step n , given by

$$\left(\prod_{i=0}^n a_i\right)^2 (x^2 + y^2 + z^2) = c^2 t^2 \quad (13)$$

and from the frame S' , the observer perceives the same

$$\left(\prod_{i=0}^n a_i\right)^2 (x'^2 + y'^2 + z'^2) = c^2 t'^2 \quad (14)$$

and we have

$$\left(\prod_{i=0}^n a_i\right)^2 (x^2 + y^2 + z^2) - c^2 t^2 = \left(\prod_{i=0}^n a_i\right)^2 (x'^2 + y'^2 + z'^2) - c^2 t'^2. \quad (15)$$

Substituting formula (12) in (15) we obtain

$$\prod_{i=0}^n a_i^2 (x^2 + y^2 + z^2) - c^2 t^2 = \prod_{i=0}^n a_i^2 \left((a_{xx}(x - vt))^2 + (a_{yy}y)^2 + (a_{zz}z)^2 \right) - c^2 (a_{tx}x + a_{tt}t)^2$$

which yields the following system

$$\begin{cases} \prod_{i=0}^n a_i^2 &= (\prod_{i=0}^n a_i^2) a_{xx}^2 - c^2 a_{tx}^2 \\ 1 &= a_{yy}^2 \\ 1 &= a_{zz}^2 \\ -c^2 &= (\prod_{i=0}^n a_i^2) a_{xx}^2 v^2 - c^2 a_{tt}^2 \\ 0 &= (\prod_{i=0}^n a_i^2) a_{xx}^2 v + c^2 a_{tx} a_{tt}. \end{cases} \quad (16)$$

The resolution of the system (16) leads to the Lorentz transformation equations T_n with a new limiting velocity

$$T_n : \begin{cases} x' = \frac{x - vt}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \frac{t - (\prod_{i=0}^n a_i^2) \frac{vx}{c^2}}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} \end{cases} \quad (17)$$

2.1 First consequence: limiting velocity

The first immediate consequence from the Lorentz transformations in an expanding universe is the following: There exists a limiting velocity for moving bodies given by

$$v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}, \quad (18)$$

which depends on the fossil light velocity (velocity of light at the beginning of the universe expansion) and the expanding parameter of the universe. Indeed, $\prod_{i=1}^n a_i$ is an increasing and bounded product greater than one, then the limiting velocity v_{l_n} is decreasing along with the space expansion of the universe. It turns out that the limiting velocity was maximal at the beginning of the expansion of the universe (step 0) and was equal to c , that is why we call it the fossil velocity of light. Thus the limiting velocity of moving body, including the light, will decrease in an expanding universe. However, the physical meaning of the equations (17) obtained in respect to moving rigid bodies and moving clocks remain the same as for the classical Lorentz transformations (deformability and lose of synchronization, time transformations, etc) except the fact that the expansion of the universe will be involved in all formulation of the physical laws as we will see later on.

2.2 Other Immediate consequences

2.2.1 Mass and Total Energy

As a consequence of these new transformations and taking into account the dynamic of the universe, we propose to adjust the Einstein's equation relative to the mass-energy as follow:

The total energy of moving body at the step n of the space expansion is given by the law

$$E_n(v) = m_n(v) \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2 \quad (19)$$

and the body's rest-mass energy at the step n of the space expansion is given by the law

$$E_n(0) = m_0 \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2, \quad (20)$$

where $m_0 = m_n(0)$ represents the mass of a body at rest relative to the observer in an expanding universe at the step n .

The relative mass at the step n is given by the law

$$m_n(v) = \frac{m_0}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}}. \quad (21)$$

Indeed, by using the Newtonian formula $\frac{dE_n(v)}{dt} = F \cdot v$, the equation (19), and the fact that the Newton's law remains valid if it is written as

$$F = \frac{d(m_n(v) v)}{dt} \quad (22)$$

give the following:

$$\frac{d\left(m_n \frac{c^2}{\prod_{i=0}^n a_i^2}\right)}{dt} = \frac{d(m_n v)}{dt} \cdot v = \frac{d(m_n v_x)}{dt} v_x + \frac{d(m_n v_y)}{dt} v_y + \frac{d(m_n v_z)}{dt} v_z. \quad (23)$$

On the other hand we have

$$\frac{d\left(m_n \frac{c^2}{\prod_{i=0}^n a_i^2}\right)}{dt} = \frac{2m_n c}{\prod_{i=0}^n a_i} \frac{d\left(m_n \frac{c}{\prod_{i=0}^n a_i}\right)}{dt} = 2m_n \frac{d\left(m_n \frac{c^2}{\prod_{i=0}^n a_i^2}\right)}{dt} \quad (24)$$

and

$$\frac{d(m_n v_s)^2}{dt} = 2m_n v_s \frac{d(m_n v_s)}{dt} \quad \text{for } s = x, y, z, \quad (25)$$

which yields in (23)

$$\frac{1}{2m_n} \frac{d\left(m_n^2 \frac{c^2}{\prod_{i=0}^n a_i^2}\right)}{dt} = \frac{1}{2m_n} \frac{d(m_n v_x)^2}{dt} + \frac{1}{2m_n} \frac{d(m_n v_y)^2}{dt} + \frac{1}{2m_n} \frac{d(m_n v_z)^2}{dt}, \quad (26)$$

then

$$\frac{d\left(m_n^2 \frac{c^2}{\prod_{i=0}^n a_i^2}\right)}{dt} = \frac{d\left(m_n^2 (v_x^2 + v_y^2 + v_z^2)\right)}{dt} = \frac{d(m_n^2 v^2)}{dt} \quad (27)$$

with $v^2 = v_x^2 + v_y^2 + v_z^2$. By integration we obtain

$$m_n^2 \frac{c^2}{\prod_{i=0}^n a_i^2} = m_n^2 v^2 + k. \quad (28)$$

For $v = 0$, and $m_n(0) = m_0$, we obtain $k = m_0^2 \frac{c^2}{\prod_{i=0}^n a_i^2}$ and then

$$m_n^2 \frac{c^2}{\prod_{i=0}^n a_i^2} = m_n^2 v^2 + m_0^2 \frac{c^2}{\prod_{i=0}^n a_i^2} \quad (29)$$

which gives the following equation

$$m_n(v) = \frac{m_0}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}}. \quad (30)$$

On the other hand the law (21) leads to the law (19), indeed, the work of displacing a body by a distance dl at the step n is

$$F dl = \frac{dP_n(v)}{dt} dl = dP_n(v) \frac{dl}{dt} = v dP_n(v) \quad (31)$$

where v is the velocity vector. If this work serves to increase the energy $E_n(v)$ of the body at the step n , then

$$dE_n(v) = v dP_n(v) = v d\left(m_n(v) v\right) = m_n(v) v dv + v^2 dm_n(v). \quad (32)$$

Since $m_n(v)$ is given by (21), we have

$$dm_n(v) = \frac{m_n(v) (\prod_{i=0}^n a_i^2) v dv}{c^2 \left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)}, \quad (33)$$

by substitution we obtain

$$dE_n(v) = m_n(v) v dv + \frac{m_n(v) (\prod_{i=0}^n a_i^2) v^3 dv}{c^2 \left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)}. \quad (34)$$

then

$$dE_n(v) = \frac{m_n(v) v dv}{\left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)}. \quad (35)$$

which gives together with formula (33)

$$dE_n(v) = \frac{c^2}{\prod_{i=0}^n a_i^2} dm_n(v). \quad (36)$$

that is to say the law (19).

The universe dynamic does not affect the mass of a given object at rest. However, object at rest has a rest energy $E_n(0)$ given by (20) in which the universe movement is manifested. When the object starts moving under any force, its total energy, momentum and mass are directly affected by the expansion parameters of the universe.

If the speed c is the fossil velocity of light, then the term $\frac{c}{\prod_{i=0}^n a_i}$ in laws (19), (20), and (21) represents the current experimental velocity of light, of course if we consider that the present time corresponds to the step n of the universe expansion, and it constitutes a limiting velocity for any motion or transfer of interaction at the step n of the universe expansion.

2.2.2 The relativistic kinetic energy

For small values of $(\prod_{i=0}^n a_i) \frac{v}{c}$, the total energy (19) goes over into the classical expression of the kinetic energy as shown by

$$E_n(v) = \frac{m_0}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2 \simeq m_0 \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2 + \frac{1}{2} m_0 v^2 + \dots \quad (37)$$

The first term represents $E_n(0)$ the energy at rest, and the second term represents the Newtonian kinetic energy. The relativistic kinetic energy E_{c_n} or the motion energy at the expansion step n can be defined by subtracting the rest energy $E_n(0)$ from the total energy $E_n(v)$

$$E_{c_n} = E_n(v) - E_n(0) \quad (38)$$

the substitution of formula (19) and (20) in the last equality gives

$$E_{c_n} = m_0 \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2 \left(\frac{1}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} - 1 \right) \quad (39)$$

and using relative mass (21) we obtain

$$E_{c_n} = (m(v) - m_0) \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2 \quad (40)$$

The kinetic energy of a body is not only related to the increase of masses when they are moving as asserted by the special relativity but also related to the universe expansion.

2.2.3 The relativistic momentum and energy

The use of equation (21) in the equation of motion (Force is equal to the rate of change of momentum) remains valid. However, here the momentum is now at the step n given by:

$$P_n(v) = m_n(v) v = \frac{m_0}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} v. \quad (41)$$

The equation $F = \frac{dP_n(v)}{dt}$ determines the motion of a body acted on by any force, but now correctly takes expanding effects into account. Clearly, the form of the momentum and force equation is very similar in relativity theory and in our approach; the effective mass $m_n(v)$ depends on the speed of motion of the body relative to the observer and the n-th step expansion parameter $\prod_{i=0}^n a_i$ according to the formula (21), while in relativity theory it is independent of the universe expansion. The crucial feature is that the effective mass $m_n(v)$ diverges as v tends to the limiting velocity $v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$ and so the momentum $P_n(v) = m_n(v)v$ also diverges. As for relativity theory, the

energy and momentum are related. Indeed, using formula (19) and (41) we obtain the following relation

$$P_n(v) c = E_n(v) \frac{v}{c} \prod_{i=0}^n a_i^2. \quad (42)$$

3 The composition of velocities

Like for special relativity we will use the classical definition of velocity. If we call \vec{u} the velocity of an object in the frame S , its components with respect to the three axes are

$$u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}. \quad (43)$$

The same velocity is measured in the frame S' as \vec{u}' with components given by

$$u'_x = \frac{dx'}{dt'}, \quad u'_y = \frac{dy'}{dt'}, \quad u'_z = \frac{dz'}{dt'}. \quad (44)$$

For simplicity and without losing generality, we will consider only that the velocity \vec{u} is parallel to the x-axes. A simple differentiation of the equations of Lorentz transformations T_n gives

$$\begin{cases} dx' = \frac{dx - v dt}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} \\ dy' = dy \\ dz' = dz \\ dt' = \frac{dt - (\prod_{i=0}^n a_i^2) \frac{v dx}{c^2}}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}}, \end{cases} \quad (45)$$

then we have

$$\begin{cases} \frac{dx'}{dt'} = \frac{dx - v dt}{dt - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} dx} \\ \frac{dy'}{dt'} = \frac{dy}{dt - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} dx} \left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)^{\frac{1}{2}} \\ \frac{dz'}{dt'} = \frac{dz}{dt - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} dx} \left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)^{\frac{1}{2}} \end{cases} \quad (46)$$

which gives

$$\begin{cases} u'_x = \frac{u_x - v}{1 - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} u_x} \\ u'_y = \frac{u_y}{1 - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} u_x} \left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)^{\frac{1}{2}} \\ u'_z = \frac{u_z}{1 - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} u_x} \left(1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}\right)^{\frac{1}{2}}. \end{cases} \quad (47)$$

Since \vec{u} is parallel to the x-axes, then the law of composition of velocity is given by

$$u' = \frac{dx'}{dt'} = \frac{u - v}{1 - (\prod_{i=0}^n a_i^2) \frac{v}{c^2} u} \quad (48)$$

and inversely

$$u = \frac{dx}{dt} = \frac{u' + v}{1 + (\prod_{i=0}^n a_i^2) \frac{v}{c^2} u'} \quad (49)$$

which has the form of the relativist law of composition of velocity with the appearance of the term $(\prod_{i=0}^n a_i)$ that characterizes the expansion of the universe at the step n .

In the extreme case where an observer in the frame S' is emitting light signals in the direction of x-axis, since the frame is in an expanding universe at the step n , then the velocity of these signals in the frame S' is given by $u' = v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$ which gives

$$u = \frac{v_{l_n} + v}{1 + (\prod_{i=0}^n a_i^2) \frac{v}{c^2} v_{l_n}} = \frac{\frac{c}{\prod_{i=0}^n a_i} + v}{1 + (\prod_{i=0}^n a_i) \frac{v}{c}} = \frac{c}{\prod_{i=0}^n a_i} = v_{l_n} \quad (50)$$

and then the limiting velocity of light v_{l_n} at the step n is invariant, that is to say the velocity of light "measured" in a moving frame at the step n , appears to be equal to $\frac{c}{\prod_{i=0}^n a_i}$ in any direction. The speed of light in vacuum, at the step n of the universe expansion, will be found the same (equal to v_{l_n}) by any two observers in uniform relative motion and this is true for the whole process of universe expansion, meanwhile this speed is decreasing together with the universe expansion.

For $n = 0$ the Lorentz transformation T_0 gives c as an invariant limiting velocity of matter, meanwhile T_1 gives $\frac{c}{a_1}$ as the invariant limiting velocity of matter, and so on, until the Lorentz transformation T_n which gives $\frac{c}{(\prod_{i=0}^n a_i)}$ as the invariant limiting velocity of matter.

It turns out that the limiting velocity of moving bodies is decreasing together with the universe expansion and that the velocity of light in empty space is independent of the velocity of its source *at each step* of the universe expansion despite of its decreasing nature together with the universe expansion. In the whole process of universe expansion the velocity of light remains independent of the reference frame of the observer.

Remark 2 *The formula (19) can be justified exactly as H. Ives [7] justified the famous formula $E = mc^2$ (Einstein [4]), by using the new composition of velocity provided by the Lorentz transformation T_n and the momentum conservation at the step n .*

4 Lorentz transformations and rest-mass of photon

The quantification of the expansion introduced above leads us to set up different steps-Lorentz transformations $(T_n)_{n \geq 0}$ that permit to quantify the physical laws evolution together with the universe expansion, and derive a relationship between the space-time views obtained by any two observers in an expanding universe. The result obtained

unifies the fundamental special-relativity results of time dilation, length contraction, and the relativity of simultaneity, in a single relation for different steps of quantified expansion. However, from these different Lorentz transformations, we sort out that the limiting velocity v_{l_n} of matter is decreasing from one step to another including the velocity of light. The velocity of light at the beginning of the expansion of the universe (step 0) was constant and equal to c an unknown value, and the T_0 -Lorentz transformation equations are given by

$$(T_0) \begin{cases} x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \end{cases} \quad (51)$$

this transformation (T_0) leads to the relativistic formulation of the physical laws

$$\begin{cases} E_0(v) & \text{the total energy at the step 0} \\ E_0(0) & \text{the rest-energy at the step 0} \\ E_{c_0}(v) & \text{the kinetic energy at the step 0} \\ P_0(v) & \text{the momentum at the step 0} \\ m_0(v) & \text{the relative mass at the step 0} \\ m_0(0) = m_0 & \text{the rest-mass at the step 0} \end{cases} \quad (52)$$

On the other hand, at the step 1 the T_1 -Lorentz transformation equations become

$$(T_1) \begin{cases} x' = \frac{x-vt}{\sqrt{1-a_1^2 \frac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \frac{t-a_1^2 \frac{vx}{c^2}}{\sqrt{1-a_1^2 \frac{v^2}{c^2}}} \end{cases} \quad (53)$$

from which we sort out that the matter has a new limiting velocity $v_{l_1} = \frac{c}{a_1}$. At this step all the equations of Newtonian mechanics hold good, and the formulations of the physical laws are obtained under some changes relative to the new limiting velocity. Similar formulations of the physical law equations (52) are introduced with the new limiting velocity and denoted by

$$\begin{cases} E_1(v) & \text{the total energy at the step 1} \\ E_1(0) & \text{the rest-energy at the step 1} \\ E_{c_1}(v) & \text{the kinetic energy at the step 1} \\ P_1(v) & \text{the momentum at the step 1} \\ m_1(v) & \text{the relative mass at the step 1} \\ m_1(0) = m_0 & \text{the rest-mass at the step 1} \end{cases} \quad (54)$$

At the step n , the T_n -Lorentz transformation equations become

$$(T_n) \left\{ \begin{array}{l} x' = \frac{x - vt}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \frac{t - (\prod_{i=0}^n a_i^2) \frac{vx}{c^2}}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}}, \end{array} \right. \quad (55)$$

in which the limiting velocity of matter reaches the value of $v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$, and where all formulations of the physical laws are possible to obtain under some changes with the hypothesis that the equations of Newtonian mechanics are valid at this step of the universe expansion (conservation of momentum, conservation of energy, etc). The new limiting velocity $v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$ at the step n induces some change in all formulations (as regards both mechanics and electro-magnetism in inertial systems), including the Maxwell equations, to obtain that the physical law remains the same under the T_n -Lorentz transformations. The formulation of the physical law equations at the step n with the new limiting velocity v_{l_n} are denoted by

$$\left\{ \begin{array}{ll} E_n(v) & \text{the total energy at the step } n \\ E_n(0) & \text{the rest-energy at the step } n \\ E_{c_n}(v) & \text{the kinetic energy at the step } n \\ P_n(v) & \text{the momentum at the step } n \\ m_n(v) & \text{the relative mass at the step } n \\ m_n(0) = m_0 & \text{the rest-mass at the step } n \end{array} \right. \quad (56)$$

where the quantities $E_n(v)$, $E_n(0)$, $E_{c_n}(v)$, $P_n(v)$ and $m_n(v)$ take the form (19), (20), (39), (41), and (21) respectively under the T_n -Lorentz transformation for all $n \geq 0$.

The Lorentz transformations (T_0) , (T_1) , ..., and (T_n) have the same mathematical form, and all the physical quantities are affected by the space expansion from one step to another, except the rest-mass m_0 which is invariant under universe expansion. Since the light has no rest frame, and since the formalism must be the same for all matter, the quantity m_0 in the above formalism represents in reality the mass of matter which is invariant under universe expansion, we will call it the *invariant mass*. The family of transformation $(T_n)_{n \geq 0}$ leads to fatal variation of the limiting velocity v_{l_n} of moving bodies according to the universe expansion. This has interesting consequences.

In the expansion process the rest energy at the step n can be evaluated compared to the rest energy at the beginning of the universe expansion. Indeed, the rest energy at the step n is given by the law

$$E_n(0) = m_0 \left(\frac{c}{\prod_{i=0}^n a_i} \right)^2 = m_0 c^2 \left(\frac{1}{\prod_{i=0}^n a_i} \right)^2, \quad (57)$$

and using the rest energy notation we obtain

$$E_n(0) = E_0(0) \frac{1}{\left(\prod_{i=0}^n a_i\right)^2}, \quad (58)$$

which means that the rest energy of matter is decreasing along with the universe expansion. However, the relative mass of matter is increasing together with the universe expansion, indeed, from the law (21) at the step n and the same law at the step $n+1$ we sort out that

$$m_{n+1}(v) \sqrt{1 - \left(\prod_{i=0}^{n+1} a_i^2\right) \frac{v^2}{c^2}} = m_n(v) \sqrt{1 - \left(\prod_{i=0}^n a_i^2\right) \frac{v^2}{c^2}} \quad (59)$$

and we obtain for all $v < \frac{c}{\prod_{i=0}^{n+1} a_i}$

$$\frac{m_{n+1}(v)}{m_n(v)} = \frac{\sqrt{1 - \left(\prod_{i=0}^n a_i^2\right) \frac{v^2}{c^2}}}{\sqrt{1 - \left(\prod_{i=0}^{n+1} a_i^2\right) \frac{v^2}{c^2}}} > 1 \quad (60)$$

which gives

$$m_{n+1}(v) > m_n(v) > \dots > m_0(v), \quad (61)$$

this means that the relative mass of matter in our universe is increasing along with the universe expansion as well as the momentum, meanwhile the rest energy is decreasing.

It is possible to determine the evolution of energy, momentum and relative mass together with the universe expansion by using the invariant mass m_0 , indeed, it is not difficult to write all these quantities at the step n function of their values at the step 0 (beginning of the expansion) and we have the law of evolution of the relative mass together with the universe expansion given by

$$m_n(v) = m_0(v) \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \left(\prod_{i=0}^n a_i^2\right) \frac{v^2}{c^2}}}, \quad (62)$$

meanwhile the evolution of momentum together with the universe expansion is given by the law

$$P_n(v) = P_0(v) \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \left(\prod_{i=0}^n a_i^2\right) \frac{v^2}{c^2}}} \quad (63)$$

and the total energy evolution together with the universe expansion is given by the law

$$E_n(v) = E_0(v) \frac{1}{\prod_{i=0}^n a_i^2} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \left(\prod_{i=0}^n a_i^2\right) \frac{v^2}{c^2}}}. \quad (64)$$

It turns out that the concept of expansion is a real source of energy, and this will be analyzed later on with a concrete example.

4.1 Quantum formalism

The velocity of light was constant at the beginning of the expansion of our universe and equal to c (an unknown fossil value). What we measure today in our experiment and what we call the velocity of light, corresponds in reality to the velocity of light at the expansion step n , where its value is given by $\frac{c}{\prod_{i=0}^n a_i}$.

The light is described in quantum mechanic as a quanta of zero mass where the relation between energy and frequency is given by the Planck-Einstein's relation $E = h\nu$ and where the momentum is given by the relation

$$P = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda_0} \quad (65)$$

where λ_0 the wave length given by

$$\lambda_0 = cT_0 = \frac{c}{\nu_0}. \quad (66)$$

The rules of translation from corpuscular terminology to wave terminology, and vice versa are based on the fact that an electromagnetic wave of length λ_0 and intensity I is considered as a stream of photons of frequency ν_0 and intensity $I = Nh\nu_0$ where N is the number of photons passing per unit time through unit areas, and the direction of motion of the wave front is the direction of motion of the photons.

In the purpose to integrate the new formalism of expansion in quantum formulation we introduce the following: We denote λ_0 the fossil wave length of the light at the beginning of the universe expansion where the celerity of photon was c , with $\nu_0 = \frac{c}{\lambda_0}$ the fossil frequency of the light at that time. We denote

$$\lambda_n = v_{l_n} T_n = \frac{\frac{c}{\prod_{i=0}^n a_i}}{\nu_n} = \frac{c}{\nu_n \prod_{i=0}^n a_i} \quad (67)$$

the current wave length of the light at the step n of the universe expansion, where the celerity of photon is $\frac{c}{\prod_{i=0}^n a_i}$, with $\nu_n = \frac{c}{\lambda_n \prod_{i=0}^n a_i}$ the light frequency at the expansion step n .

Hence the wave length represents a distance, therefore by using the property of the expansion of the space the wave length expands and we have $\lambda_n = \lambda_0 \prod_{i=0}^n a_i$ which leads to the following relation between the fossil light frequency and the light frequency at the expansion step n

$$\nu_n = \frac{c}{\lambda_n \prod_{i=0}^n a_i} = \frac{c}{\lambda_0 \prod_{i=0}^n a_i^2} \quad (68)$$

and then

$$\nu_n = \nu_0 \frac{1}{\prod_{i=0}^n a_i^2}. \quad (69)$$

From the relativistic relation between energy and momentum $P_o(v)c = E_o(v)\frac{v}{c}$, we sort out the momentum of photon with velocity $v = v_{l_n}$ given by

$$P_0(v_{l_n}) = \frac{E_0(v_{l_n})}{c} \frac{1}{\prod_{i=0}^n a_i} \neq 0, \quad (70)$$

thus $E_0(v_{l_n}) \neq 0$ and well defined for $v = v_{l_n}$, then

$$E_0(v_{l_n}) = \frac{E_0(0)}{\sqrt{1 - \frac{1}{\prod_{i=0}^n a_i^2}}} \neq 0, \quad (71)$$

which gives that the rest energy of a photon $E_0(0)$ is not equal to zero.

The substitution of (71) in the formula (70) gives

$$P_0(v_{l_n}) = \frac{E_0(0)}{c \prod_{i=0}^n a_i} \frac{1}{\sqrt{1 - \frac{1}{\prod_{i=0}^n a_i^2}}}. \quad (72)$$

The precedent equality can be expressed using the rest energy (58) at the step n and v_{l_n} as

$$P_0(v_{l_n}) = \left(\frac{\prod_{i=0}^n a_i}{c} \right) \left(\frac{E_0(0)}{\prod_{i=0}^n a_i^2} \right) \frac{1}{\sqrt{1 - \frac{1}{\prod_{i=0}^n a_i^2}}}, \quad (73)$$

therefore the energy and momentum of photon are related by the law

$$P_0(v_{l_n}) = \frac{E_n(0)}{v_{l_n}} \frac{1}{\sqrt{1 - \frac{1}{\prod_{i=0}^n a_i^2}}}. \quad (74)$$

The law (74) leads to the following approximation

$$P_0(v_{l_n}) \simeq \frac{E_n(0)}{v_{l_n}} \quad (75)$$

where $v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$ is the limiting velocity of light measured at the step n. The relation (75) was introduced experimentally by A. H. Compton in 1923, [3].

4.2 Photon mass

Using the formulation given by Lorentz transformation (T_0), the relative mass of moving body is given by

$$m_0(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{for } \forall v < c \quad (76)$$

which gives

$$m_0 = m_0(v) \sqrt{1 - \frac{v^2}{c^2}}. \quad (77)$$

then

$$m_0^2 = m_0^2(v)(1 - \frac{v^2}{c^2}) \iff m_0^2 c^2 = m_0^2(v)(c^2 - v^2), \quad (78)$$

which gives

$$m_0^2 c^4 = m_0^2(v) c^4 - m_0^2(v) c^2 v^2, \quad (79)$$

then for all $v < c$ we have

$$m_0^2 c^4 = E_0^2(v) - P_0^2(v) c^2. \quad (80)$$

Nevertheless, if the invariant mass m_0 of a photon was zero at the origin of the expansion when its velocity was c , it will remain zero at the step n when the light move at speed $v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$ because of its natural invariance under the universe expansion.

If we suppose that the invariant mass of a photon $m_0 = 0$ at the step n , $\forall n > 0$, (since for $n = 0$ the formula (80) is not defined for $v_{l_0} = c$) from the universe expansion, then the speed of light is given by $v = v_{l_n}$, and by substitution in the the formula (80) we obtain

$$0 = E_0^2(v_{l_n}) - P_0^2(v_{l_n}) c^2. \quad (81)$$

Using the value of $P_0(v_{l_n})$ given by formula (72), the equality (81) becomes

$$0 = E_0^2(v_{l_n}) - \frac{E_0^2(0)}{\prod_{i=0}^n a_i^2} \left(\frac{1}{1 - \frac{1}{\prod_{i=0}^n a_i^2}} \right). \quad (82)$$

On the other hand we have (71), and then equality (82) gives

$$E_0^2(0) \left(\frac{1}{1 - \frac{1}{\prod_{i=0}^n a_i^2}} \right) = \frac{E_0^2(0)}{\prod_{i=0}^n a_i^2} \left(\frac{1}{1 - \frac{1}{\prod_{i=0}^n a_i^2}} \right), \quad (83)$$

which is equivalent to

$$E_0^2(0) = \frac{E_0^2(0)}{\prod_{i=0}^n a_i^2}. \quad (84)$$

Since the rest energy of a photon at the beginning of the universe expansion (step 0) was not null, then formula (84) leads to the contradiction

$$1 = \frac{1}{\prod_{i=0}^n a_i^2} \quad \forall n > 0 \quad \text{which is impossible,} \quad (85)$$

then the invariant mass of photon $m_0 \neq 0$.

Along the universe expansion the invariant mass m_0 of matter remains the same, meanwhile the associated rest energy decreases ($E_0(0) > E_1(0) > \dots > E_n(0)$), and the momentum of matter increases. If the rest energy of photon was zero, it can not decrease.

Since $\forall v < c$, $E_0(v)\sqrt{1 - \frac{v^2}{c^2}} = m_0 c^2$, then we can evaluate the invariant mass m_0 for a photon of velocity $v = v_{l_n}$. Indeed, we have

$$m_0 = \frac{E_0(v_{l_n})}{c^2} \sqrt{1 - \frac{1}{\prod_{i=0}^n a_i^2}}, \quad (86)$$

the use of the formula (71) in the precedent equality gives

$$m_0 = \frac{E_0(0)}{c^2} \quad (87)$$

and we have

$$m_0 = \frac{E_0(0)}{c^2} = \left(\frac{\prod_{i=0}^n a_i^2}{c^2} \right) \left(\frac{E_0(0)}{\prod_{i=0}^n a_i^2} \right) \quad (88)$$

using (58) and the velocity of light v_{l_n} at the step n to obtain

$$m_0 = \frac{E_0(0)}{c^2} = \frac{E_n(0)}{v_{l_n}^2}, \quad (89)$$

which means that the ratio between rest energy and velocity of light is invariant under universe expansion. The formula (87) means that the light has non zero invariant mass, and with non zero mass there must exist some gravity. This new fact invites to revise the interpretation of the deviation of the light rays near massive corp [6]. Photons are affected by gravity and their normally straight trajectories are bent because of gravity as simple as that. However, the invariant mass of a photon depends on the fossil rest energy (the rest energy at the step n is impossible to determine since the light is never at rest). We know only that the invariant mass and the rest energy of a photon are not null.

4.3 The conversion of the rest-energy into kinetic energy

In the purpose to sort out the real nature of energy gained by matter in an expanding universe we introduce the following example of inelastic collision:

Consider two identical particles which move toward each other along straight line, with equal speeds, and equal invariant mass m_0 , they collide and stick together in an expanding space at the step n. The conservation of momentum at the step n gives:

$$m_n(v) v - m_n(-v) v = M_n(V) V \quad (90)$$

thus

$$0 = M_n(V) V \quad (91)$$

which gives $V = 0$, so the final object is at rest with mass $M_n(0)$.

The conservation of total energy at the step n from the universe expansion gives

$$m_n(v) \frac{c^2}{\prod_{i=0}^n a_i^2} + m_n(-v) \frac{c^2}{\prod_{i=0}^n a_i^2} = M_n(V) \frac{c^2}{\prod_{i=0}^n a_i^2}, \quad (92)$$

since the final object is at rest (that is to say $V = 0$), then it follows that

$$m_n(v) + m_n(-v) = 2m_n(v) = M_n(0), \quad (93)$$

then we obtain

$$M_n(0) = \frac{2m_0}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}}, \quad (94)$$

the mass of final object is larger than the sum of the original masses.

To evaluate the nature of energy created with the increase of mass together with the universe expansion from the step 0 to the step n we have to compare the same collision at the origin of the expansion (step 0) and at the step n. Indeed the universe expansion creates the increase of moving masses as

$$M_n(0) - M_0(0) = 2m_o \left(\frac{1}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right). \quad (95)$$

For a small value of $(\prod_{i=0}^n a_i) \frac{v}{c}$ we can approximate (95) as

$$M_n(0) - M_0(0) \simeq 2m_o \left(1 + \frac{1}{2} \left(\prod_{i=0}^n a_i^2 \right) \frac{v^2}{c^2} - 1 - \frac{1}{2} \frac{v^2}{c^2} \right) \quad (96)$$

which gives

$$M_n(0) - M_0(0) \simeq \frac{1}{2} (2m_o) \left(\prod_{i=0}^n a_i^2 \right) \frac{v^2}{c^2} \left(1 - \frac{1}{\prod_{i=0}^n a_i^2} \right). \quad (97)$$

The gain of masses represents a rest energy at the step n given by

$$\left(M_n(0) - M_0(0) \right) \frac{c^2}{\prod_{i=0}^n a_i^2} \simeq \frac{1}{2} (2m_o) v^2 \left(1 - \frac{1}{\prod_{i=0}^n a_i^2} \right). \quad (98)$$

this means that the rest energy gained by a moving particles together with the universe expansion is equal approximatively to the sum of the classical kinetic energy of the two particles factor the rate of increase of rest energy due to the universe expansion, that is to say $\left(1 - \frac{1}{\prod_{i=0}^n a_i^2} \right)$. Indeed, from formula (58) it is easy to evaluate the variation of the rest energy between the beginning of the universe expansion (step 0) and the universe expansion at the step n, we obtain

$$E_0(0) - E_n(0) = E_0(0) \left(1 - \frac{1}{\prod_{i=0}^n a_i^2} \right) \quad (99)$$

from which it turns out that $\left(1 - \frac{1}{\prod_{i=0}^n a_i^2}\right)$ represents the rate of decrease of the rest energy together with the universe expansion.

Since the mass of final object is larger than the sum of the original masses, we can approximate this excess of mass for a small value of $(\prod_{i=0}^n a_i)^{\frac{v}{c}}$, and compare it to the same value at the beginning of the universe expansion. Indeed the approximation of the law (94), for all $n \geq 0$, gives

$$M_n(0) \simeq 2m_0 + \frac{1}{2}(2m_0)\left(\prod_{i=0}^n a_i^2\right)\frac{v^2}{c^2} \quad (100)$$

which gives

$$\left(M_n(0) - 2m_0\right) \simeq \frac{1}{2}(2m_0)\left(\prod_{i=0}^n a_i^2\right)\frac{v^2}{c^2}, \quad (101)$$

and then we obtain

$$\left(M_n(0) - 2m_0\right) \simeq \left(M_0(0) - 2m_0\right) \prod_{i=0}^n a_i^2. \quad (102)$$

It turns out that the excess mass of the composite object at the step n of the universe expansion compared to the mass of the same composite object at the beginning of the expansion (step 0) is increasing together with the universe expansion and then the kinetic energy brought in also increases.

4.4 Fission of atoms in an expanding space

Suppose that we have an atom of uranium with rest mass $M_n(0) = M$ measured in an expanding universe at the step n, and suppose that something happens so that the atom flies into two equal pieces moving with speed v , so that each part has a relative mass $m_n(v)$ at the step n. Suppose that these pieces encounter enough material to slow them up until they stop and then each part will have an invariant mass $m_n(0) = m_0$. To reach its rest position each part will give an amount of energy left in the material in some form, whatever. The left energy for one part is then given by

$$E_{left} = \left(m_n(v) - m_n(0)\right) \frac{c^2}{\prod_{i=0}^n a_i^2}. \quad (103)$$

The conservation of total energy at the universe expansion step n gives

$$E_n(0) = 2E_n(v) \iff M = 2m_n(v), \quad (104)$$

so the total liberated energy at the step n is given by

$$E_{L_n} = 2E_{left} = 2\left(m_n(v) - m_0\right) \frac{c^2}{\prod_{i=0}^n a_i^2} = \left(M - 2m_0\right) \frac{c^2}{\prod_{i=0}^n a_i^2}, \quad (105)$$

$$E_{L_n} = (M - 2m_0) \frac{c^2}{\prod_{i=0}^n a_i^2}, \quad (106)$$

which can be written as

$$E_{L_n} = E_n(v) - E_n(0), \quad (107)$$

where $\forall v < \frac{c}{\prod_{i=0}^n a_i}$, $E_n(v) = 2m_n(v) \frac{c^2}{\prod_{i=0}^n a_i^2}$.

The law given by (106) for $n = 0$ was used to estimate how much energy would be liberated under fission in the atomic bomb since the mass of uranium atom was known as well as the atoms into which it splits (iodine, xenon, and so on). However, it turns out from the law (106) that the released energy when an atom of uranium undergoes fission is correlated to the universe expansion which has interesting consequences.

As the universe expands we have the law (61), which gives

$$2(m_n(v) - m_0) > 2(m_{n-1}(v) - m_0) > \dots > 2(m_0(v) - m_0), \quad (108)$$

and from (106) we have

$$E_{L_n} \prod_{i=0}^n a_i^2 > E_{L_{n-1}} \prod_{i=0}^{n-1} a_i^2 > \dots > E_{L_0}. \quad (109)$$

Using the law (99) we obtain

$$\frac{E_{L_n}}{E_n(0)} > \frac{E_{L_{n-1}}}{E_{n-1}(0)} > \dots > \frac{E_{L_0}}{E_0(0)}, \quad (110)$$

this means that the ratio between the total liberated energy by an atom of uranium and its rest energy is increasing together with the universe expansion.

The total liberated energy by an atom of uranium (107) can be approximated, indeed, we have

$$E_{L_n} = E_n(v) - E_n(0) = E_n(0) \frac{1}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} - E_n(0), \quad (111)$$

then we have

$$E_{L_n} = E_n(0) \left(\frac{1}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} - 1 \right), \quad (112)$$

by using the law (58) that expresses the correlation between the rest energy at the step n and the rest energy at the step 0, we obtain

$$E_{L_n} = E_0(0) \frac{1}{\prod_{i=0}^n a_i^2} \left(\frac{1}{\sqrt{1 - (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}}} - 1 \right). \quad (113)$$

For a small value of $(\prod_{i=0}^n a_i) \frac{v}{c}$, we obtain the following approximation

$$E_{L_n} \simeq E_0(0) \frac{1}{\prod_{i=0}^n a_i^2} \left(\frac{1}{1 - \frac{1}{2} (\prod_{i=0}^n a_i^2) \frac{v^2}{c^2}} - 1 \right). \quad (114)$$

which gives

$$E_{L_n} \simeq E_0(0) \frac{1}{\prod_{i=0}^n a_i^2} \left(1 + \frac{1}{2} \left(\prod_{i=0}^n a_i^2 \right) \frac{v^2}{c^2} + \left(\frac{1}{2} \right)^2 \left(\prod_{i=0}^n a_i^2 \right)^2 \frac{v^4}{c^4} - 1 \right). \quad (115)$$

then

$$E_{L_n} \simeq E_0(0) \left(\frac{1}{2} \frac{v^2}{c^2} + \left(\frac{1}{2} \right)^2 \left(\prod_{i=0}^n a_i^2 \right) \frac{v^4}{c^4} \right), \quad (116)$$

it turns out that the total liberated energy by an atom of uranium is increasing together with the universe expansion and we have for one atom of uranium the following evolution of liberated energy from one step to another

$$E_{L_n} > E_{L_{n-1}} > \dots > E_{L_0}, \quad (117)$$

This has interesting consequences that will change our understanding of gravity, the missing masses, or the estimation of stars energy and others. The energy in our universe increases together with the universe expansion, and this excess of energy is naturally due to the recession of galaxies from each other. The increase of energy and mass in our universe compensates the increase of distance between matters and might explain why the gravity exists at long distance. Our estimation of stars energy, or galaxies masses are erroneous if we omit to take into account the movement of matter due to the universe expansion. We conclude by raising the fatal fact that there is no stationary safe distance when an atom of uranium undergoes fission in an expanding universe, since the liberated energy increases along with the universe expansion, the security of nuclear reactor must be endlessly reconsidered. It turns out that the tame of this energy is strewn with risk and peril together with the universe expansion.

4.5 The experimental data and the new quantification

The family of transformations $(T_0), (T_1), (T_2), \dots, (T_n)$ is a mathematical formulation of the Lorentz transformation that incorporates the given quantification of the space expansion. This family of transformations $(T_i)_{i \geq 0}$ is defined for all $v < v_{l_n} = \frac{c}{\prod_{i=0}^n a_i}$, where v_{l_n} represents the limiting velocity of moving bodies at the expansion step n . From this limiting velocity it turns out that the velocity of light is decreasing as the universe expands, and the limiting velocity v_{l_n} was maximal only at the step 0 (in the beginning of the universe expansion $v_{l_0} = c$), however, at each step the velocity of light is constant in the sense that it is independent of the reference frame of the observer.

All formulations and laws based on the assumption that the velocity of light in empty space is independent of the velocity of its source remain valid for matter and related physical phenomenon during the universe expansion step by step, and the relativistic formulation of the physical laws remains valid *experimentally* since the velocity of light measured experimentally represents in reality $\frac{c}{\prod_{i=0}^n a_i}$ (in different formulations). At each step of the universe expansion the measured velocity of light remains invariant from one frame to another in uniform relative motion.

All our experimental tests and applications of the special relativity laws at present time are and remain valid since the substitution of the experimental measure of the light speed in the T_0 -Lorentz transformation will represent T_n -Lorentz transformations. Only the future and the past are erroneous because of the variation of the velocity of light together with the universe expansion. Nevertheless some corrections must be added to the principle of relativity to fits the new quantification of the universe expansion:

The principle of relativity states that *all laws of nature are the same in all inertial system of coordinates*, we have to add "at each step of the universe expansion". The principle states that not a single physical experiment could discover special properties for one of the inertial systems *at each step* of the universe expansion. All inertial systems are equivalent at the step n for all $n \geq 0$.

The second postulate pertains to *the constancy of the velocity of light in a vacuum for all inertial systems*. We have to add "at each step of the universe expansion". From this it follows that the velocity of light in the "receding" and "approaching" directions must be the same *at each step* of the universe expansion, that is to say the velocity of light is independent of the light source and measuring instruments at each step of the universe expansion. However the velocity of light varies (more precisely decreases) together with the universe expansion.

We close this article by some comments on the notion of time. The notion of relative time as for special relativity remains valid with some adjustment, from the metric (9) is clear that time is a required coordinate to specify a physical event and that this quantity can not be defined without the notion of space and the notion of expansion, the dynamic of the universe is involved in the definition of time. The expansion of the universe affects the time.

If we refer to the proper time τ as the time which is measured by an observer in a reference frame at the step n of the universe expansion in which the events occur at the same spatial point, then $dx = dy = dz = 0$ and the metric (9) gives

$$d\sigma_n^2 = c^2 d\tau_n^2, \quad (118)$$

meanwhile in another inertial reference frame the same events will verify (9), which gives

$$c^2 d\tau_n^2 = c^2 dt_n^2 - \left(\prod_{i=0}^n a_i \right)^2 (dx^2 + dy^2 + dz^2), \quad (119)$$

from which we obtain

$$\left(\frac{d\tau}{dt_n} \right)^2 = 1 - \prod_{i=0}^n a_i^2 \frac{v^2}{c^2} \quad (120)$$

where $v^2 = \left(\frac{dx}{dt_n} \right)^2 + \left(\frac{dy}{dt_n} \right)^2 + \left(\frac{dz}{dt_n} \right)^2$. It turns out that the rate of proper time for a system varies with motion through space as for special relativity, and what is new here is that the rate of the proper time varies together with the universe expansion even if v is constant.

If we refer to the universe proper time τ as the time which is measured by an observer in a reference frame at rest at the step n of the universe expansion in which

the events occur at the same spatial point, it is not difficult to extract from the metric (9) the relationship between proper time and coordinate time

$$d\tau = dt_n \sqrt{1 - \prod_{i=0}^n a_i^2 \frac{v^2}{c^2}}. \quad (121)$$

The time depends on the observer and the universe expanding step. What is new here is to associate space, time and expansion in the definition of event. Any change of reference or steps affects all of them. If the proper time τ of the universe is the same from one step to another and if the velocity v and c are constant from one step to another then it turns out from the step $n - 1$ to the step n that

$$1 = \frac{dt_n}{dt_{n-1}} \frac{\sqrt{1 - \prod_{i=0}^n a_i^2 \frac{v^2}{c^2}}}{\sqrt{1 - \prod_{i=0}^{n-1} a_i^2 \frac{v^2}{c^2}}} \quad (122)$$

and then

$$\frac{dt_n}{dt_{n-1}} = \frac{\sqrt{1 - \prod_{i=0}^{n-1} a_i^2 \frac{v^2}{c^2}}}{\sqrt{1 - \prod_{i=0}^n a_i^2 \frac{v^2}{c^2}}} > 1 \quad (123)$$

which allows us to compare the coordinate time between successive steps as

$$dt_n > dt_{n-1} \quad (124)$$

which means that the clocks at the step n will run slower than an identical type of clocks at the previous step, the time will run slower together with the universe expansion.

One more thing, the product $\prod_{i=0}^n a_i$ which allows the quantification of the universe expansion was extracted from the fractal manifold model [1]. More details about a natural construction of $\prod_{i=0}^n a_i$ and the universe expanding nature can be found in [2].

References

- [1] Ben Adda, F, Mathematical Model for Fractal Manifold, *International journal of Pure and Applied Mathematics*, Volume 38, **N 2**, P.159-190, 2007.
- [2] Ben Adda, F, Expansion and hidden dimensions in a new cosmological model, arXiv:0710.5387, Nov 2007.
- [3] Compton, A, A quantum theory of the scattering of X-rays by light elements, *Physical Review* 21, 483-502, 1923.
- [4] Einstein A, Does the inertia of body depend upon its energy content?, translated from "Ist die trägheit eines körpers von seinem energiegehalt abhängig?", *Annalen der physik*, 17, 1905.

- [5] Einstein A, Kosmologische betrachtungen zur allgemeinen relativitätstheorie, *Preussische Akademie Der Wissenschaften, Sitzungsberichte*, P142-152, 1917.
- [6] Eddington, A, Monthly notices of the royal astronomical society, 80, p96, 1919.
- [7] Ives, H, Derivation of the mass energy relationb, J. Opt. Soc. Americ, 42, 8, 540-543, 1952.